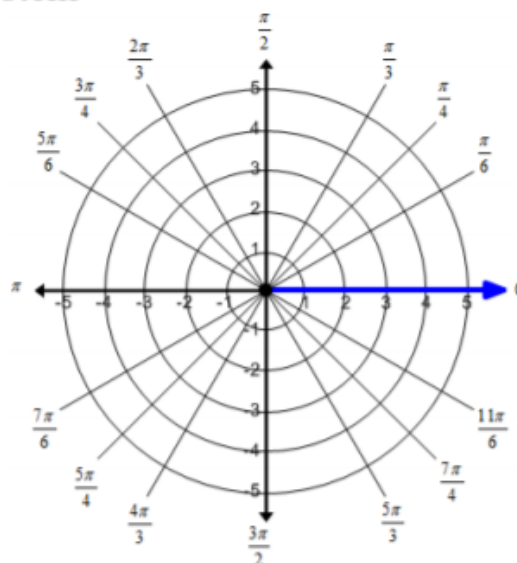


8.3 Graphing Polar Equations

Graphing Equations on the Polar Coordinate System

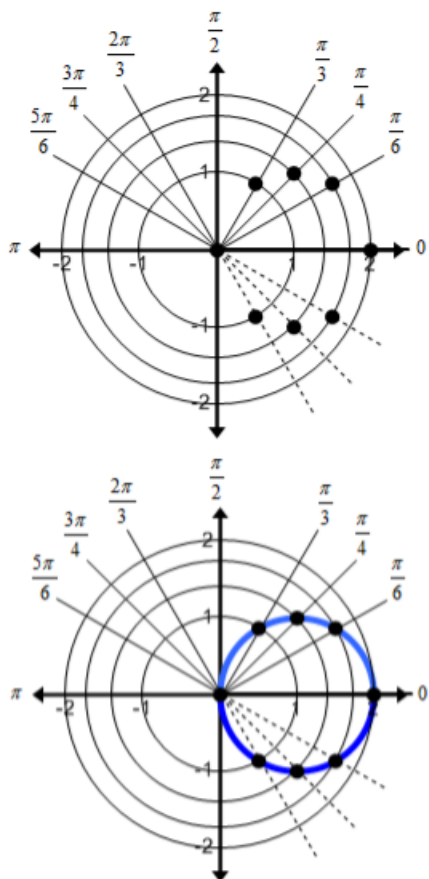
Graphing on the polar coordinate system is like graphing on a set of concentric circles whose center is the pole. The independent variable is θ and the dependent variable is r . The graph is made by plotting all the points that are solutions to the equation.

The graph at the right shows lines going through the pole at angle values for which exact trigonometric values are known. Using these angles makes it easier to graph the polar equations by hand.



Example:

Graph the equation $r = 2 \cos \theta$.



Evaluate the equation at each value of θ then plot the points on the polar coordinate system.

θ	$r = 2 \cos \theta$
0	2
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-1
$\frac{3\pi}{4}$	$-\sqrt{2}$
$\frac{5\pi}{6}$	$-\sqrt{3}$
π	-2

Connect the points in the order that they were plotted.

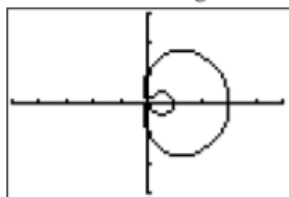
Types of Polar Graphs

Limacons

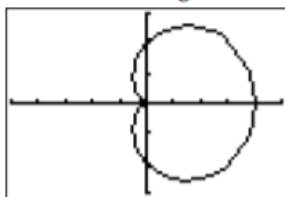
Equations: $r = a \pm b \sin \theta$ or $r = a \pm b \cos \theta$ where $a > 0$, $b > 0$

Graphs:

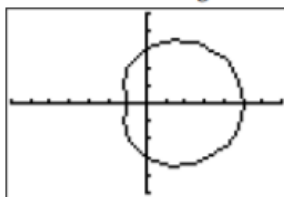
Inner Loop $\frac{a}{b} < 1$



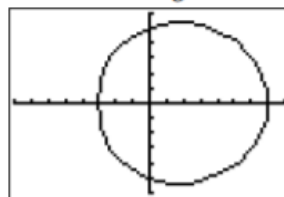
Cardioid $\frac{a}{b} = 1$



Dimpled $1 < \frac{a}{b} < 2$



Convex $\frac{a}{b} \geq 2$

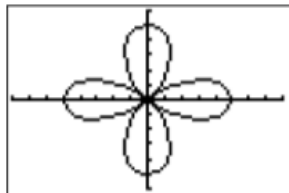


Rose Curves

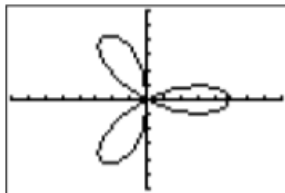
Equations: $r = a \sin(n\theta)$ or $r = a \cos(n\theta)$ where $n > 1$ is an integer and $a \neq 0$. The number of petals is determined by the value of n . If n is even then there will be $2n$ petals. If n is odd then there will be n petals.

Graphs:

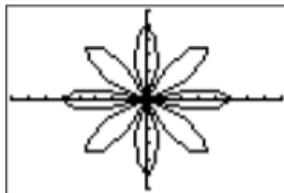
$n = 2$



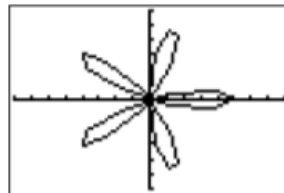
$n = 3$



$n = 4$



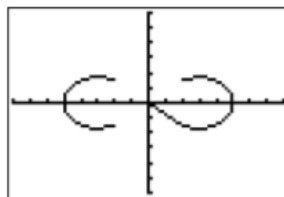
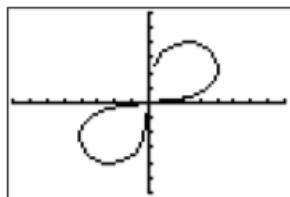
$n = 5$



Lemniscates

Equations: $r^2 = a^2 \sin 2\theta$ or $r^2 = a^2 \cos 2\theta$, $a \neq 0$

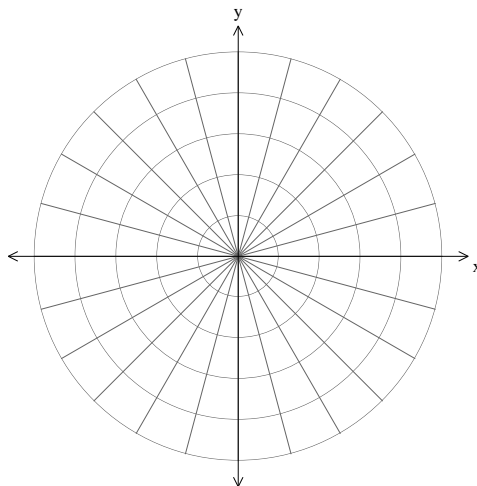
Graphs:



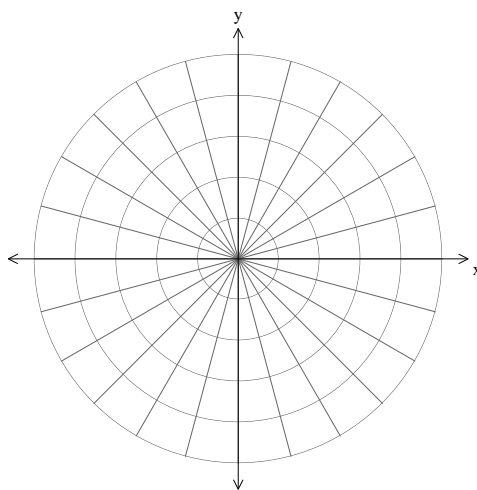
Graphing Polar Equations

Examples: Sketch the graphs of the following:

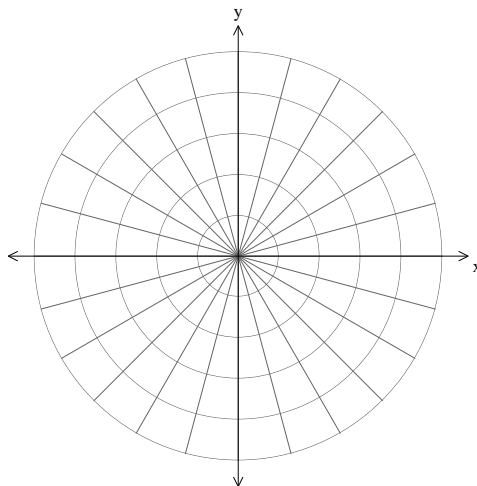
a) $r = 4 \sin \theta$



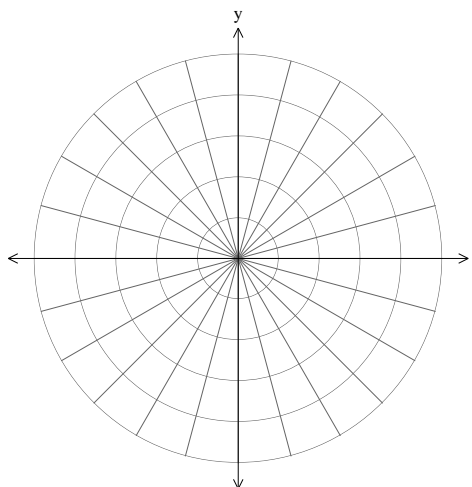
b) $r = 3 \cos(2\theta)$



c) $r = 2 + 3 \sin \theta$



d) $r^2 = 4\sin(2\theta)$



Use technology to graph the equation $r = 4\sin(3\theta)$.

First you will need to change the mode on your calculator so that you are in polar mode (make sure that radians is selected). Push

QUIT
MODE

then arrow down to FUNC PAR POL SEQ. Use your

arrow keys to arrow over to POL and push

ENTRY/SOLVE
ENTER

so that polar is selected. Exit the screen by pushing

2ND

QUIT
MODE

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi r<^>θ
FULL HORIZ G-T
4NEXT4
```

Next you will need to set the window for your equation. Push

TABLE **F2**
WINDOW

. There are a few new items when graphing in polar mode. Typically θ will need to have a minimum of 0 and a maximum of 2π . Set the x minimum to -7 and the x maximum to 7. Set the y minimum to -5 and the y maximum to 5. Exit the

screen by pushing

2ND

QUIT
MODE

```
WINDOW
θmin=0
θmax=6.2831853...
θstep=1/1308996...
Xmin=-7
Xmax=7
Xscl=1
↓Ymin=-5
```

Enter the equation by pushing

STAT PLOT **F1**
Y=

. The variable button

LINK
X,T,θ,n

will now be θ . Once you have entered the equation,

push

TABLE **F5**
GRAPH

and you should get something that looks like the graph at the right.

```
Plot1 Plot2 Plot3
✓r1=4sin(3θ)
✓r2=
✓r3=
✓r4=
✓r5=
✓r6=
```

